TRINITY COLLEGE

Mathematics Methods U3/4
Test 3 2022

Section 1 Calculator Free
Discrete Random Variables

STUDENT'S NAME ______________________________

DATE: Monday 9th May **TIME:** 15 minutes **MARKS:** 14

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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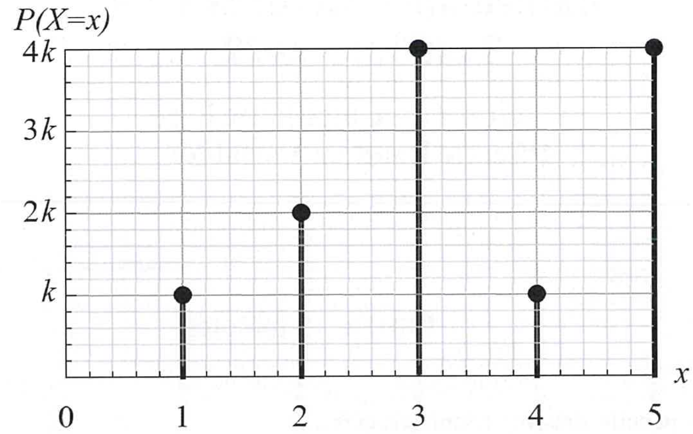
1. (7 marks)

The discrete random variable X can take the values 1, 2, 3, 4, 5. The probability distribution for X is shown graphically below.



Determine:

(a) the value of k. [2]

$12k = 1$ ✓ creates eq.

$\therefore k = \frac{1}{12}$ ✓ final ans.

(b) $P(X \geq 3)$ [1]

$9/12$ ✓

(c) $P(X = 3 \mid X \geq 3)$ [2]

$4/9$ ✓

(d) the expected value. [2]

$1 \times \frac{1}{12} + 2 \times \frac{2}{12} + 3 \times \frac{4}{12} + 4 \times \frac{1}{12} + 5 \times \frac{4}{12}$ ✓ uses formula

$= \frac{41}{12}$ ✓ final solution

2. (7 marks)

(a) A discrete random variable X, where $X = 0, 1, 2, 3, 4, 5, 6, 7$ has a uniform distribution.

(i) Determine the expected value and variance of X. [3]

$E(X) = (1+2+3+4+5+6+7)\frac{1}{8}$

$= \frac{7}{2}$ ✓ $E(X) = \frac{7}{2}$

$Var(X) = E(X^2) - [E(X)]^2$ Eq + sub ✓

$(1^2+2^2+3^2+4^2+5^2+6^2+7^2)\frac{1}{8} - \left(\frac{7}{2}\right)^2$

$\frac{70}{4} - \frac{49}{4}$

$= \frac{21}{4}$ ✓ solution for $Var(X)$

(ii) A discrete random variable is defined by $Y = 5 - 2X$. Calculate the expected value and variance of Y. [2]

$E(Y) = 5 - 2\left(\frac{7}{2}\right)$

$= -2$ ✓

$V(X) = (-2)^2 \frac{21}{4}$

$= 21$ ✓

(b) A discrete uniform distribution Z, has outcomes 0 to n and have an expected value, $E(X) = 5.5$. Calculate the value of n. [2]

$\frac{n}{2} = 5.5$ ✓ eq.

$\therefore n = 11$ ✓ solution.

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Mathematics Methods U3/4
Test 3 2022

Section 2 Calculator Assumed
Discrete Random Variables

STUDENT'S NAME ______________________________

DATE: Monday 9th May **TIME:** 35 minutes **MARKS:** 34

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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3. (6 marks)

Tokens numbered 1 to 20 are placed in a bag, and one is selected at random:

- Let $X = 1$ if a prime number is selected, and $X = 0$ otherwise.
- Let $Y = 1$ if a number greater than 8 is selected, and $Y = 0$ otherwise.

(a) Determine the probability of success of X and Y respectively. [2]

$P(X) = 8/20$ ✓ $P(X)$

$P(Y) = 12/20$ ✓ $P(Y)$

(b) Calculate the mean and variance of X. [1]

$\mu(x) = 0.4$

✓ both solutions

$\sigma^2 = 0.24$

(c) Calculate the mean and variance of Y. [1]

$\mu(x) = 0.6$

✓ both solutions

$\sigma^2 = 0.24$

(d) Compare the standard deviation of X and Y and justify why this occurred? [2]

They are the same. ✓ states same.

Probability of $X = 1 - Y$ ✓ expl.

4. (9 marks)

It is known the probability of a bread roll ~~pen~~ being below the satisfactory weight for sale in a large batch is 0.12. At the bakery, bread rolls are sold in packets of 6.

(a) Describe the probability distribution function. [2]

X: The number of bread rolls that are below the satisfactory weight. ✓ describe

$X \sim B(6, 0.12)$ ✓ write in Binomial Dist

(b) Determine $E(X)$ and st.dev(X) [2]

$E(X) = 6 \times 0.12$

$= 0.72$ ✓ E(X)

$St.dev(X) = \sqrt{6 \times 0.12 \times 0.88}$

$= 0.7960$ ✓ std(x)

(c) Determine the probability that

(i) A randomly selected packet has greater than 2 bread rolls that are below the satisfactory weight for sale. [1]

$P(X > 2) = 0.0261$ ✓ solution.

(ii) that there were no more than 1 bread roll that is below the satisfactory weight for sale in each packet if 6 packets were randomly selected. [2]

$P(X \leq 1) = 0.8444$ ✓ $P(X \leq 1)$

$(0.8444)^6 = 0.3625$ ✓ x^6

(iii) in a large order of 20 packets of bread rolls that no more than 3 of these have greater than 2 bread rolls that are below the satisfactory weight for sale. [2]

$Y \sim B(20, 0.0261)$ ✓ writes dist.

$P(Y \leq 3) = 0.9984$ ✓ $P(Y \leq 3)$

5. (10 marks)

A slot machine is programmed to operate at random, making various payouts after patrons pay \$2 and press a start button. The random variable X is the amount of the payout, in dollars, in one play of the machine. Each payout can be assumed to be independent of other payouts.

The probability, P, that the machine makes a certain payout, x, is shown in the table below.

Payout (\$) x	0	1	2	5	10	20	50	100
Probability $P(X = x)$	0.25	0.45	0.2125	0.0625	0.0125	0.005	0.005	0.0025

(a) Determine the probability that

(i) in one play of the machine, a payout of more than \$1 is made. [1]

$P(X > 1) = 1 - (0.25 + 0.45)$

$= 0.3$ ✓

(ii) in ten plays of the machine, it makes a payout of \$5 no more than once. [2]

$Y \sim B(10, 0.0625)$ ✓ write dist.

$P(Y \leq 1) = 0.8741$ ✓ $P(Y \leq 1)$.

(iii) in five plays of the machine, the second payout of \$1 occurs on the fifth play. [3]

$Z \sim B(4, 0.45)$ ✓ write dist

$P(Z = 1) = 0.2995$ ✓ calc $P(Z = 1)$

$P = 0.2995 \times 0.45$

$= 0.1348$ ✓ soln.

(b) Calculate the mean and standard deviation of X. [2]

$\mu = 1.9125$ $\sigma_x = 6.321$

✓ μ ✓ σ_x

(c) In the long run, what percentage of the patron's money is returned to them? [2]

$\frac{1.9125}{2} \times 100 = 95.625\%$

✓ Fraction ✓ percentage.

6. (9 marks)

In the mailroom of a large company, it is known that 20% of incoming letters contain an invoice. Let X be the number of randomly chosen letters that are opened until an invoice is discovered.

(a) Complete the table below for the values of $x = 1, 2, 3,$ and 4. [2]

x	1	2	3	4
$P(X = x)$	0.2	0.16	0.128	0.1024

✓ 2 prob correct

✓ all prob correct

(b) Determine the rule for $P(X = x)$ for any integer value greater than 0. [2]

$P(X=x) = 0.8^{x-1} \times 0.2$

✓ 0.8^{x-1} ✓ 0.2

(c) Calculate

(i) $P(X = 10)$ [1]

$= 0.8^9 \times 0.2 = 0.0268$ ✓ sln.

(ii) $P(3 \le X \le 6)$ [2]

✓ calc $x=5$, $x=6$

$0.128 + 0.1024 + 0.8^4 \times 0.2 + 0.8^5 \times 0.2$

$= 0.3778$ ✓ calc total.

(iii) the smallest value of k, so that $P(X = k) < 0.001$. [2]

$0.2 \times 0.8^{x-1} < 0.001$ ✓ write inequality

$x < 24.74$

$k = 25$ ✓ soln.